Maple Program for Studying Physics Phenomena with Applications in Civil Engineering

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Summary

Nowadays, students need adequate software to study Physics in universities and to have high levels of educational achievement and decent test scores. As our experience has demonstrated along the years, the combination of the experimental physics and computational methods motivates students to explore deeper the world of physics phenomena in a modern way. Also, they test their knowledge showing what they have learned.

This paper presents how physics phenomena are really well understood by the students because of using a powerful algebra system like Maple 17. Also, the Maple 17 program it is a useful tool for solving physics problems. The Graphics package of Maple 17 program has been used for the applications at the laboratories and the seminars of the Physics I and Physics II courses that are designated to the first and second semesters, respectively at the Faculty of Civil Engineering and Building Services of “Gheorghe Asachi” Technical University.

The versatility and the wide range of applications of Maple 17 make it a powerful tool in assisting the students for solving problems and drawing graphs. This algebra system also provides many applications in engineering. Also, a platform as Maple presents advantages like speed, flexibility and many graphical facilities. For this reasons we consider the Maple program to be a reliable tool for teaching and learning physics.

In our work we present some examples of plotting the displacement and speed for damped systems. The plot command that creates a two-dimensional plot and the plots[animate] command that generates an animation are both used for making the graphs in the case of overdamped, critically damped and underdamped systems, respectively. Using this powerful algebra system the students will work faster and a deeper understanding of the differences between the three cases of damped systems will be developed.

KEYWORDS: Maple 17, Physics, Civil Engineering, physics phenomena, damped oscillations.
1. INTRODUCTION

Computational instruments are more and more widely used and many teachers deal with them to improve the teaching-learning process [1]-[5]. The experience in this area demonstrated that traditional methods in Physics and engineering together with computational physics are an important tool that gives to the students a deeper understanding of the physical phenomena and engineering studies [6]-[10]. These tools motivate and enable the students to improve their learning process and obtain better results to the exams. A plethora of programs like Maple, Mathematica and Matlab [11]-[13] are used for teaching Physics.

The authors use Maple 17 [11] to improve Civil Engineering physics teaching-learning. Among other many useful commands the software provides the commands used to perform calculations, to solve differential equations, to solve equations, to make plots and to produce plots with animations.

We have implemented both Maple and Mathematica programs in the Physics course, laboratories and seminars. Regularly, we improve our computational methods using the newest versions of these algebra systems. Also, the students can work in campus and achieve a better understanding of the physics phenomena and change ideas.

In this paper we focus our attention to the use of Maple 17 program for plotting some examples of physics phenomena that are studied in the chapter Damped Oscillations. We present the plots and plots[animate] commands for the cases of overdamped, critically damped and underdamped systems. The graphs of displacement and speed are plotted.

2. MAPLE 17 APPLICATIONS FOR DAMPED OSCILLATIONS

2.1. Applications of Maple 17 for damped systems – the plots for the displacement and speed

Because in the last decade we have gained experience in using several computational programs and developing simulations of physics phenomena [14]-[17] we decided to implement the newest algebra systems in the Physics course, laboratories and seminars.

As we pointed out above, the students are motivated to use the Maple 17 program because it provides them a high-level programming language plots and animations. They have used the program also for other calculations. Maple 17 allows to perform Numeric and Symbolic Computations, Visualizations, Matrix Computations, Differential Equations, Control Systems Design, Dictionary of Math and Engineering, Units and Tolerances, Maplets and MapleNet, Smart Documents.
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Special Functions, Optimization, Statistics, Programming, Code Generation, and OpenMaple and Education and Assessment.

2.2.1. Plotting 2D of the displacement for damped oscillations
Firstly, we briefly present the theory for damped oscillations. In this case upon the oscillator act the elastic force and the friction. Writing the second law of dynamics and solving the differential equation for the displacement three different cases are obtained, the case of overdamped systems, critically damped systems and underdamped oscillations, respectively.

The second order linear differential equation with constant coefficients is given by

\[ \frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + \omega_0^2 x = 0, \]  

where \( \frac{\gamma}{m} = 2\delta \) and \( \omega_0^2 = \frac{k}{m} \) and with \( \gamma \) the damping constant, \( k \) the spring constant and \( \omega_0 \) the natural frequency that represents the frequency at which the oscillator oscillates in the absence of any friction. The case of oscillation without friction is the case of harmonic oscillations. In the presence of damping the solution of equation (1) which is the displacement will behave differently depending on the size of \( \gamma \) (\( \delta^2 \)) relative to \( \omega_0^2 \). Solving the characteristic equation

\[ r^2 + 2\delta r + \omega_0^2 = 0, \]  

the solutions are

\[ r_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}. \]  

There are three cases:

1. Overdamped system \( \delta > \omega_0 \), the roots of the characteristic equation (2) are real and negative and the displacement is

\[ x(t) = C_1 e^{\delta t} + C_2 e^{-\delta t}, \]  

where \( C_1 \) and \( C_2 \) are two arbitrary constants that are determined from the initial conditions (displacement and speed at \( t = 0 \)). This is the case of aperiodic motion and the mass crosses equilibrium position at most once. There are no oscillations.

2. Critically damped system \( \delta = \omega_0 \) in this case the roots of the characteristic equation (2) are equal and the displacement is

\[ x(t) = C e^{-\delta t}. \]  

This is an incomplete mathematical solution because the solution of (1) must have two arbitrary constant. Physically, these constants allow the setting of the initial conditions \( x(t) \) and \( v(t) \) at \( t = 0 \). The complete solution of equation (1) is

\[
x(t) = (C_1 + C_2 t) e^{-\delta t}.
\]

This is the case of critically aperiodic motion and the mass also crosses equilibrium position at most once. There are no oscillations, the body tends towards the equilibrium position without oscillates around it.

3. Underdamped oscillations \( \delta < \omega_0 \), the characteristic equation (2) has two complex conjugate roots and the displacement is

\[
x(t) = A_0 e^{-\delta t} \sin(\omega t + \varphi),
\]

with the amplitude \( A(t) = A_0 e^{-\delta t} \) decreasing exponentially overtime. The system oscillates quasi periodic with the quasi frequency \( \omega = \sqrt{\omega_0^2 - \delta^2} \). The mass can cross the equilibrium position infinitely often (excepting the zero solution).

In the following, we present the plots for the displacement in the cases of the overdamped, critically damped and underdamped systems, respectively.

In the case of the overdamped system the equation (1) has the expression

\[
\frac{d^2 x}{dt^2} + 10 \frac{dx}{dt} + 9 x = 0.
\]

The initial conditions for the displacement and for the speed are \( x(0) = 1 \) and \( \frac{dx}{dt}(0) = 3 \). Solving the characteristic equation and applying the initial conditions the constants \( C_1 \) and \( C_2 \) are determined. Their values are \( C_1 = \frac{3}{2} \) and \( C_2 = -\frac{1}{2} \).

The solution is given by

\[
x(t) = \frac{3}{2} e^{-t} - \frac{1}{2} e^{-9t}.
\]

The Maple commands for plotting 2D are with(plots); plot(\( \frac{3}{2} e^{-t} - \frac{1}{2} e^{-9t}, t=0..10 \)). The graph is given in Figure 1.
In the case of the critically damped system the equation (1) has the expression

\[ \frac{d^2 x}{dt^2} + 4 \frac{dx}{dt} + 4x = 0. \]

The initial conditions are expressed by \( x(0) = 2 \) and \( \frac{dx}{dt}(0) = 5 \). In this case, also solving the characteristic equation and with the aid of the initial conditions the constants \( C_1 \) and \( C_2 \) are \( C_1 = 2 \) and \( C_2 = 9 \). The displacement is given by

\[ x(t) = (2 + 9t)e^{-2t}. \]  

The commands are `with(plots); plot((2+9t)*exp(-2*t), t=0..10)` and the plot is given in Figure 2.
In Figure 2 is given the graph of displacement for the underdamped oscillations. In this case, for the equation (7) that is given by the expression \( x(t) = A_0 e^{-\alpha t} \sin(\omega t + \varphi) \) the expressions of \( A_0 \), \( \varphi \), \( a \) and \( b \) are given by \( A_0 = \sqrt{a^2 + b^2} \), \( \tan \varphi = \frac{a}{b} \), \( A_0 \sin \varphi = a \), \( A_0 \cos \varphi = b \). In this case \( A_0 \) represents the constant value of the amplitude and \( \varphi \) the initial phase.

In Figure 3 is given the graph of displacement for the underdamped oscillations. In this case, for the equation (7) that is given by the expression \( x(t) = A_0 e^{-\alpha t} \sin(\omega t + \varphi) \) the expressions of \( A_0 \), \( \varphi \), \( a \) and \( b \) are given by \( A_0 = \sqrt{a^2 + b^2} \), \( \tan \varphi = \frac{a}{b} \), \( A_0 \sin \varphi = a \), \( A_0 \cos \varphi = b \). In this case \( A_0 \) represents the constant value of the amplitude and \( \varphi \) the initial phase.

The values obtained after the calculations for \( A_0 \) and \( \varphi \) are \( A_0 = 2 \) and \( \varphi = \frac{\pi}{4} \).
The displacement is given by

$$x(t) = 2e^{-0.194t} \sin(18.25t + \frac{\pi}{4}).$$  \hspace{1cm} (10)

The Maple commands for underdamped oscillations are with (plots);

\begin{verbatim}
plot(2*exp(-0.194*t)*sin(18.25*t + Pi/4), t=0..10).
\end{verbatim}

In Figure 4 we make the plots of the displacement for all the three cases of the overdamped system, the critically damped system and the underdamped oscillations on the same graph, respectively.
The Maple commands are with (plots); \[ f := \frac{3}{2} e^{-t} - \frac{1}{2} e^{-gt}; \quad g := (2 + 9t) e^{-2t}; \]
\[ h := 2e^{-0.194t} \sin(18.25t + \frac{\pi}{4}); \]
plot(\{f(t), g(t), h(t), t=0..10\}).

Figure 4. Displacements for the overdamped system, the critically damped system and the underdamped oscillations

In the case of the underdamped oscillations the larger the damping factor the quasi frequency becomes smaller. The quasi-period becomes longer. If the damping
factor becomes smaller for its zero value the case of harmonic oscillations is reached.

Using the command animate, after this click on Animation and Play we have the possibility to modify the values of the damping constant. In Figure 5 we present the case of underdamped oscillations. The commands are with(plots); animate(plot,[2e^{-bt} \sin(18.25t + \frac{\pi}{4}), t=0..10],b=0..10).

Figure 5. Displacement of underdamped oscillations with animation

2.2.2. Plotting 2D of the speed for damped oscillations

We plot the speed in the case of the underdamped oscillations given by equation (10).

The expression for the speed in the case of the underdamped oscillations is
\[ v(t) = \omega A_0 e^{-\delta t} \left[ -\frac{\delta}{\omega} \sin(\omega t + \varphi) + \cos(\omega t + \varphi) \right]. \] (11)

For \( \frac{\delta}{\omega} \ll 1 \) the first term from equation (11) can be neglected and we obtain for the speed

\[ v(t) = \omega A_0 e^{-\delta t} \cos(\omega t + \varphi). \] (12)

The graph of the speed is given in Figure 6. The Maple commands are with(plots); plot(36.5e^(-0.194t)\cos(18.25t + \frac{\pi}{4}), t=0..10).

Figure 6. Speed of an underdamped oscillation
3. CONCLUSIONS

The Maple 17 provides new functions and packages presenting in this way many advantages. Very important are the facilities for calculus, plotting and animation. We present some examples for an important chapter of Physics the one of damped oscillations. In all oscillatory systems there is damping. As applications of damping we notice that all the vibrating systems are connected to damping because they are dissipative systems [18]. A small damping has a little influence on the natural frequencies, but the damping play an important role in limiting the amplitude of the oscillator at resonance. Structural engineering is based on damping, in earthquake areas or on structural components that are exposed to wind gusts. In the case of tall buildings that are placed in regions with seismic risk some damping systems are built in to help protect the buildings. For the cable-stayed bridges, in order to prevent the oscillations produced by wind with high speeds, hydraulic dampers have to be installed at the end of the cables.

We consider that working at Physics laboratories and solving problems with the aid of computer programs is a main part of the teaching-learning process and our students should understand the computational process. In this view, we point out that the students are more and more interested in the algebra computer systems for learning. Using Maple 17 the students will improve their abilities of working at a high-level at Physics laboratories and seminars. They can also take advantage of the computing programs and use the computers in campus for plotting and solve problems. As an immediate perspective, we want to introduce more applications of the Maple program and in this way to provide to the students the environment to work in the best conditions.

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