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Optimal seismic response control of a long span cable-stayed bridge for a benchmark problem

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Summary

Structural Control has been growing in a fast pace as a subject contributing with means to alleviate the effects of harming loads. Correlated with other domains of human activity (electronics, automatic control, computer science, robotics, new materials, etc) important changes in philosophy and practice are recorded in Civil Engineering.

Because Structural Control is still an expensive approach to protect structures, in the views are structures as long cable-supported bridges and tall buildings that are vital for people and social activities, especially during and after strong winds or earthquakes.

This paper is showing a developing procedure of the classical method of optimal control. A first step into this development was previously done when the first author was showing that it is possible to lower the degree of arbitrariness for the coefficients in the weighting matrices based on energy considerations.

In this work the method is further developed when the attention is given to using reduced state models, which is a more realistic approach than using the full-state method as in previous works.

For the FEM model of the cable-stayed bridge given by an international benchmark, simulations have been performed using the method described above. External actions are three important strong earthquake acceleration records. The discrete time approach time and time-delay existing between the calculation and application of the control forces are taken into consideration. Also the process noise and measurements are considered. A simple predictive procedure is proposed.

Results of the application are showing that the time-response and also frequency responses are considerably reduced: stresses, bending moments, forces, displacements, velocities, and accelerations are kept into allowable limits. Based on the benchmark performance indexes the controlled system is very competitive.

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1. INTRODUCTION

Structural Control has been growing in a fast pace as a subject contributing with means to alleviate the effects of harming loads since its beginning [1]. Last years have shown a stress on researches about active control of large structures with many active devices. Optimal control has been used in [2] and [3] in a centralized setting. Decentralized controllers have been proposed in [4], [5] and [6]. Also, sliding mode control has been analyzed as a method to cope with uncertainties [7].

In the context of optimal control, an energy-based method for choosing the weighting parameters was developed [8]. The method is very convenient because it implies a simple way to set the control parameters. Using this method, it is possible to control large structures using many devices and, in this way, the efficiency and reliability of the control are highly increased. Then other authors adopted similar energy-based control strategies [9].

The international benchmark control problem for seismic response of cable-stayed bridges is used [10] to prove the efficiency of the proposed strategy, For this goal, the full state methodology for choosing the weighting control matrices is set up in order to adapt it to the needs of canonical modal transformation and model reduction procedures [11].

Therefore, realistic simulations with few measurement devices and reduced order estimator are performed. Simulations take into account the discrete-time aspects of a real application, along with process noise, measurement noise, and application of control forces time delay. In this paper, the method is further improved and tested through the use of a simple predictive method for the measurements, in order to avoid time delays. Comparisons of the predictive strategy and non-predictive control with the benchmark sample control strategy are done. Results show good behavior of the proposed control methodologies according to a set of evaluation criteria established by the benchmark.

2. METHODOLOGY. OPTIMAL CONTROL APPROACH FOR REDUCED STATE SYSTEMS

Optimal active control is a time domain strategy that is appropriate for controlling the response of structures subjected to strong earthquakes, [12]. The strategy allows minimizing the induced structural energy [13].

In Structural Control, the state equation of motion for a *n* degree of freedom controlled system under seismic action is:

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$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{h}\ddot{x}_g(t), \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

where $\mathbf{x}(t)$ is the $2n$-dimensional state, and $\mathbf{A}$, $\mathbf{B}$ are appropriate matrices.

Setting the control actions as $\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t)$, the goal of the method is to obtain the feedback gain matrix $\mathbf{K}$ to minimize a performance index J defined by

$$J = \frac{1}{2}\int_0^\infty \left[\mathbf{x}'(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}'(t)\mathbf{R}\mathbf{u}(t)\right] dt \tag{2}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices, $2n\times2n$-dimensional and $m\times m$-dimensional, respectively; and m is the number of the actuators. Minimization of the performance criterion (2) implies to solve the Riccati equation

$$\mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}'\mathbf{P} + \mathbf{A}'\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{3}$$

Then, the control gain matrix is $\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}'\mathbf{P}$.

Appropriate settings can be found for the full states-based matrices $\mathbf{Q}$ and $\mathbf{R}$. For example, if $\mathbf{Q} = \mathrm{diag}(\mathbf{K}_1, \mathbf{M}_1)$, then the first term in the brackets of Equation (2) is an energy expression and therefore Equation (2) leads to minimization of the energy of the structural response. Matrix $\mathbf{R}$ can be set as $\mathbf{R} = r\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and r is a scalar, the unique parameter to be determined [13].

Usually, only few measurements are available. In this case, the output of the system, $\mathbf{y}$, is expressed through a second equation complementing the state-space model, in the form:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{h}\ddot{x}_g \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases} \tag{4}$$

where $\mathbf{C}$ is a $p\times2n$ measurement matrix and $\mathbf{D}$ is the $p\times m$ matrix showing the influence of the control forces on the output.

A first step to avoid the inconvenience of using all states is to apply a canonical transformation $\mathbf{x}_c = \mathbf{T}_c\mathbf{x}$ or $\mathbf{x} = \mathbf{T}_c^{-1}\mathbf{x}_c = \mathbf{P}_c\mathbf{x}_c$ based on the eigenvector matrix $\mathbf{P} = \mathbf{T}_c^{-1}$. This way, the system (4) takes the new form:

$$\begin{cases} \dot{\mathbf{x}}_c = \mathbf{A}_c\mathbf{x}_c + \mathbf{B}_c\mathbf{u} + \mathbf{h}_c\ddot{x}_g \\ \mathbf{y} = \mathbf{C}_c\mathbf{x}_c + \mathbf{D}\mathbf{u} \end{cases} \tag{5}$$

where $\mathbf{A}_c = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$, $\mathbf{B}_c = \mathbf{P}_c^{-1}\mathbf{B}$, $\mathbf{C}_c = \mathbf{P}_c^{-1}\mathbf{C}$, and $\mathbf{h}_c = \mathbf{P}_c^{-1}\mathbf{h}$.

The second step, based on controllability and observability gramians [14,15], is to use a state coordinate transformation matrix $\overline{\mathbf{T}}$ applied to the system (5), i.e., $\overline{\mathbf{x}} = \overline{\mathbf{T}}\mathbf{x}_c$. The resulting system is a balanced system:

$$\begin{cases} \dot{\overline{\mathbf{x}}} = \overline{\mathbf{A}}\overline{\mathbf{x}} + \overline{\mathbf{B}}\mathbf{u} + \overline{\mathbf{h}}\ddot{x}_g \\ \mathbf{y} = \overline{\mathbf{C}}\overline{\mathbf{x}} + \mathbf{D}\mathbf{u} \end{cases} \tag{6}$$

where $\overline{\mathbf{A}} = \overline{\mathbf{T}}\mathbf{A}_c\overline{\mathbf{T}}^{-1}$, $\overline{\mathbf{B}} = \overline{\mathbf{T}}\mathbf{B}_c$, $\overline{\mathbf{C}} = \mathbf{C}_c\overline{\mathbf{T}}^{-1}$, and $\overline{\mathbf{h}} = \overline{\mathbf{T}}\mathbf{h}_c$. If only the first most significant q states are retained for the structural response, the system (6) can be rewritten in the form:

$$\begin{cases} \begin{Bmatrix} \dot{\overline{\mathbf{x}}}_1 \\ \mathbf{0} \end{Bmatrix} = \begin{bmatrix} \overline{\mathbf{A}}_{11} & \overline{\mathbf{A}}_{12} \\ \overline{\mathbf{A}}_{21} & \overline{\mathbf{A}}_{22} \end{bmatrix} \begin{Bmatrix} \overline{\mathbf{x}}_1 \\ \overline{\mathbf{x}}_2 \end{Bmatrix} + \begin{Bmatrix} \overline{\mathbf{B}}_1 \\ \overline{\mathbf{B}}_2 \end{Bmatrix}\mathbf{u} + \begin{Bmatrix} \overline{\mathbf{h}}_1 \\ \overline{\mathbf{h}}_2 \end{Bmatrix}\ddot{x}_g \\ \mathbf{y} = \begin{bmatrix} \overline{\mathbf{C}}_1 & \overline{\mathbf{C}}_2 \end{bmatrix} \begin{Bmatrix} \overline{\mathbf{x}}_1 \\ \overline{\mathbf{x}}_2 \end{Bmatrix} + \mathbf{D}\mathbf{u} \end{cases} \tag{7}$$

where $\overline{\mathbf{x}}_1$ are the states to be retained, a q-dimensional vector. $\overline{\mathbf{x}}_2$ are the states to be eliminated, $\overline{\mathbf{x}}_2 = -\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{A}}_{21}\overline{\mathbf{x}}_1 - \overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{B}}_2\mathbf{u} - \overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{h}}_2\ddot{x}_g$. This way, the reduced state system is:

$$\begin{cases} \dot{\mathbf{x}}_r = \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u} + \mathbf{h}_r\ddot{x}_g \\ \mathbf{y} = \mathbf{C}_r\mathbf{x}_r + \mathbf{D}_r\mathbf{u} + \mathbf{h}_y\ddot{x}_g \end{cases} \tag{8}$$

where:, $\mathbf{A}_r = \overline{\mathbf{A}}_{11} - \overline{\mathbf{A}}_{12}\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{A}}_{21}$, $\mathbf{B}_r = \overline{\mathbf{B}}_1 - \overline{\mathbf{A}}_{12}\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{B}}_2$, $\mathbf{h}_r = \overline{\mathbf{h}}_1 - \overline{\mathbf{A}}_{12}\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{h}}_2$, $\mathbf{C}_r = \overline{\mathbf{C}}_1 - \overline{\mathbf{C}}_2\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{A}}_{21}$, $\mathbf{D}_r = \mathbf{D} - \overline{\mathbf{C}}_2\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{B}}_2$, $\mathbf{h}_y = -\overline{\mathbf{C}}_2\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{h}}_2$, and $\mathbf{x}_r = \overline{\mathbf{x}}_1$.

Therefore, for the system (8), the index to be minimized is:

$$J = \frac{1}{2}\int_0^{t_f} \left[\mathbf{x}_r'\mathbf{Q}_e\mathbf{x}_r + \mathbf{u}'(\mathbf{R} + \mathbf{R}_e)\mathbf{u} + 2\mathbf{x}_r'\mathbf{N}_e\mathbf{u}\right]dt \tag{9}$$

where $\mathbf{A}_e = \left[\mathbf{I} \quad -\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{A}}_{21}\right]'$, $\mathbf{B}_e = \left[\mathbf{0} \quad -\overline{\mathbf{A}}_{22}^{-1}\overline{\mathbf{B}}_2\right]'$, $\mathbf{N}_e = \mathbf{A}_e'\left(\overline{\mathbf{T}}^{-1}\right)'\mathbf{P}_c'\mathbf{Q}\mathbf{P}_c\overline{\mathbf{T}}^{-1}\mathbf{B}_e$ $\mathbf{Q}_e = \mathbf{A}_e'\left(\overline{\mathbf{T}}^{-1}\right)'\mathbf{P}_c'\mathbf{Q}\mathbf{P}_c\overline{\mathbf{T}}^{-1}\mathbf{A}_e$, and $\mathbf{R}_e = \mathbf{B}_e'\left(\overline{\mathbf{T}}^{-1}\right)'\mathbf{P}_c'\mathbf{Q}\mathbf{P}_c\overline{\mathbf{T}}^{-1}\mathbf{B}_e$.

Note that, because of the above transformations, Equation (9) is an approximation of the Equation (2). The corresponding Riccati equation is then:

$$\mathbf{PA} - (\mathbf{PB} + \mathbf{N}_e)(\mathbf{R} + \mathbf{R}_e)^{-1}(\mathbf{B}'\mathbf{P} + \mathbf{N}_e') + \mathbf{A}'\mathbf{P} + \mathbf{Q}_e = \mathbf{0} \tag{10}$$

and the gain matrix is expressed by: $\mathbf{K} = (\mathbf{R} + \mathbf{R}_e)^{-1}(\mathbf{B}'\mathbf{P} + \mathbf{N}_e')$.

For real applications, the model from Equation (8) can be adapted into the form:

$$\begin{cases} \dot{\mathbf{x}}_r = \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u} + \mathbf{h}_r\ddot{x}_g + \mathbf{G}\mathbf{w} \\ \mathbf{y}_m = \mathbf{C}_r\mathbf{x}_r + \mathbf{D}_r\mathbf{u} + \mathbf{h}_y\ddot{x}_g + \mathbf{H}\mathbf{w} + \mathbf{v} \end{cases} \tag{11}$$

where **w** and **v** are the process noise and the measurement noise vectors respectively; **G** and **H** are distribution matrices and $\mathbf{y}_m$ is the measured output vector.

The states of the system (11) can be estimated from the measured outputs using for example a Kalman filter [16]:

$$\dot{\hat{\mathbf{x}}}_r = \mathbf{A}\hat{\mathbf{x}}_r + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y}_m - \mathbf{C}_r\hat{\mathbf{x}}_r - \mathbf{D}_r\mathbf{u}) \tag{12}$$

where $\hat{\mathbf{x}}_r$ is the vector of estimated states and **L** is the filter gain matrix deduced from solving also a Riccati equation. Then the control forces are: $\mathbf{u} = -\mathbf{K}\hat{\mathbf{x}}_r$.

All the procedure shown above is formulated in continuous time. The application in the next section is using the discrete time version of the method as the practice requires. A time delay between the computation of control forces and their application is also considered. Supposing a time delay equal to the sampling time, Δt, the control forces are applied at the time, t_{i+1}. This is one step after the real measuring time, t_i, when the measurement vector, $\mathbf{y}_{m,i}$, was obtained.

Also, a very simple linear predictive scheme can be applied: the current measurement vector, $\mathbf{y}_{m,i}$, is considered an average from the previous one, $\mathbf{y}_{m,i-1}$, and the future one, $\mathbf{y}_{m,i+1}$, i.e.

$$\mathbf{y}_{m,i+1} = 2\mathbf{y}_{m,i} - \mathbf{y}_{m,i-1} \tag{13}$$

This way, the estimated states from Equation (12) are deduced based on the predicted measurement vector defined by Equation (13).

3. APPLICATION TO THE BENCHMARK MODEL

The control procedure described previously is applied to the model of a cable-stayed bridge, Cape Girardeau over Mississippi River, that is the object of a benchmark problem [10]. The main span of the bridge is 350.6 m with the side spans of 142.7 m in length. It has a total of 128 cables that connect the 29.3 m wide deck with the towers, 100 m and 105 m tall. Figure 1 shows the bridge FEM model (left) and the three applied actions, El-Centro NS, 1940, Mexico City, 1985, and Gebze NS, 1999, (right).

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Statical reduction of the initial FEM model leads to a 419 dynamical degrees of freedom system. The system is reduced to a system with 30 states, as it is done in the benchmark sample scheme, for reasons of comparisons.

For the digital implementation, continuous to discrete-time signal converters are included. Four longitudinal direction accelerometers are placed on the on tops of the towers and one is located in the mid span. Four sensors measuring displacements were located between the towers and the deck [10].

In order to evaluate and compare the results of the proposed control strategies, the benchmark establishes 18 performance criteria. First six criteria refer to peak responses; the criteria from seven to eleven are related to normed responses, while the criteria twelve to eighteen are concerned to control strategy.

In order to choose a suitable value for the weighting matrix **R**, based on an unique scalar *r*, comparisons between the responses or criteria heve been performed. For this application, in the case of non-predictive control option, the scalar *r* took 14 different values within the interval [1.0e19, 1.0e22]. In the case of predictive strategy, the scalar *r* took 16 values in the same interval.

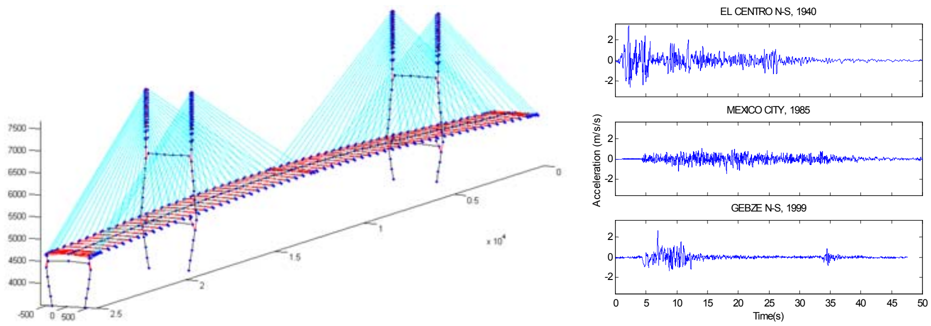

Fig.1. FEM model of the bridge (left) and the three external actions (right)

As an example of the design strategy, in Figure 2 the variation of the criteria number three (relative, maximum overturning moment for towers) as a function of the scalar *r* are presented. The values of these criteria given in the benchmark sample solution are also presented (as horizontal lines of the same type) for comparison. It can be seen a non-constant behavior of the results (values of criteria). In addition, this behavior is still a function of the three external actions (earthquakes), so different in content and effects.

Following the previous paragraphs' ideas, Table 1 shows a numerical comparison for the different results obtained for the first 15 criteria under the selected values for *r*. For each of the three earthquake actions there are shown three different columns with criteria values. The case *a)* refers to the benchmark given sample control (as the base of comparison). The case *b)* refers to applying the strategy

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proposed in the section above without prediction, with r = 3.0e20, while *c)* refers to the case using prediction with r = 7.5e20.

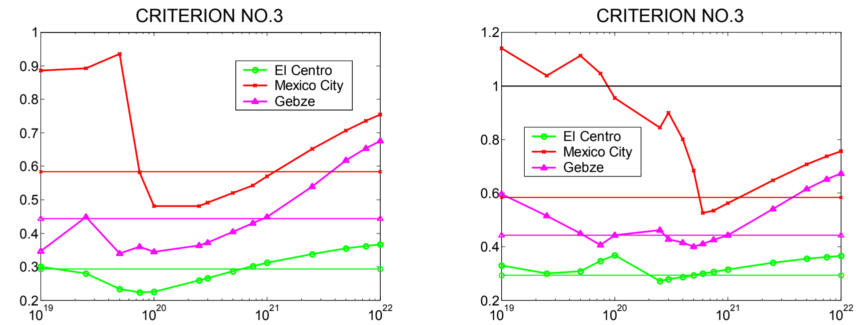

Fig.2. Criterion no.3 for the cases without/with prediction (left/right)

From Table 1, important observations can be withdrawn.

Table 1. Comparison of the benchmark (a) and the obtained results (b,c)

Crit.	No-ctrl.	El Centro Earthquake			Mexico Earthquake			Gebze Earthquake		
		a)	b)	c)	a)	b)	c)	a)	b)	c)
J_1	1.0	0.3868	0.3517*	0.3764**	0.4582	0.4366**	0.4201*	0.4540	0.4061*	0.4102**
J_2	1.0	1.0681	1.0213*	1.0534**	1.3693	1.2048*	1.3030**	1.3784	1.2262*	1.2532**
J_3	1.0	0.2944**	0.2673*	0.3066	0.5836	0.4918*	0.5345**	0.4434	0.3725*	0.4260**
J_4	1.0	0.6252	0.5883*	0.5625**	0.6140**	0.5786*	0.6422	1.2246*	1.2673**	1.6362
J_5	.8029 .1481 .3832	0.1861*	0.1944**	0.2045	0.0775	0.0694*	0.0740**	0.1481*	0.1629**	0.1991
J_6	1.0000	1.2006**	1.1730*	1.3164	2.3317*	2.3472**	3.1808	3.5640*	3.8828**	5.3403
J_7	1.0000	0.2257	0.2118*	0.2177**	0.3983	0.3649*	0.3695**	0.3231	0.2986*	0.3009**
J_8	1.0000	1.1778	1.0617*	1.1333**	1.2118	1.0795*	1.1615**	1.4371**	1.4026*	1.6618
J_9	1.0000	0.2665	0.2401*	0.2599**	0.4192**	0.3875*	0.4209	0.4552**	0.4520*	0.5246
J_{10}	1.0000	0.8813**	0.8056*	0.8946	1.1067**	1.0863*	1.2066	1.4570*	1.7621**	2.2947
J_{11}	0.0867 0.0225 0.0423	0.0280	0.0245*	0.0272**	0.0103	0.0092*	0.0100**	0.0171**	0.0167*	0.0194
J_{12}	0.0000	0.0016*	0.0019**	0.0016*	0.0006*	0.0006*	0.0005**	0.0017**	0.0018	0.0015*
J_{13}	0.0000	0.7883**	0.7702*	0.8643	1.1742*	1.1820**	1.6018	1.9541*	2.1288**	2.9279
J_{14}	0.0000	0.0027*	0.0027*	0.0020**	0.0018	0.0016*	0.0011**	0.0074	0.0071**	0.0058**
J_{15}	0.0000	0.0004**	0.0004**	0.0003*	0.0002**	0.0002**	0.0001*	0.0007**	0.0007**	0.0006*
Total 1st pos.		3 (20%)	12 (80%)	2 (13%)	3 (20%)	11 (73%)	2 (13%)	5 (34%)	7 (47%)	2 (13%)
Total 2nd pos.		5 (33%)	3 (20%)	8 (54%)	4 (27%)	4 (27%)	8 (54%)	5 (33%)	7 (47%)	5 (33%)
Total 3rd pos.		7 (47%)	0 (0%)	5 (33%)	8 (53%)	0 (0%)	5 (33%)	5 (33%)	1 (6%)	8 (54%)

* Best value (first position); ** Second position.

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Therefore the superiority of the non-predictive strategy is strongly shown, specially by As a general idea, the strategy *b)* (non-predictive) looks the most successful while the predictive strategy *c)* is better than the benchmark example, *a)*. The explanation for this is that the prediction was acceptable for the displacement sensors and not acceptable for the acceleration sensors due to the high dynamics of these sensors' signals and the relatively long sampling time (0.02 sec.). The first 6 criteria, i.e. reduction of the maximum responses for base shear, shear at deck level, overturning moment, moment at the deck level, cable tension, displacement at the abutment level.

4. CONCLUSIONS

In this paper a previous work, [11], is further improved to the need of more realistic applications. The weighting matrix **Q** choice is done on energy-based procedure for the full state system. Since the measurements and the estimators cannot assure the knowledge or approximation of too many states, a reduced order model is employed [14,15]. The matrix **Q** is reduced following similar transformations, as the system itself. Simple prediction scheme for measurements is proposed, in order to avoid delays in applying control forces. A finite element model of a bridge proposed as a benchmark problem for structural control under seismic actions [10] is used for application. Simulations take into account the discrete-time aspects of a real application, along with process noise and measurement noise. Good behavior of the controlled system according to the benchmark evaluation criteria set, especially for the case without prediction, is noted and discussed.

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