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Homogeneity on Designing the Structural Systems

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Summary

The paperwork presents the analysis over the effects of the variation of constitutive materials' characteristics in the elastic designing of the structural systems. It has been analyzed the transformation of the actions into sectional efforts using the given techniques by the mechanic materials and static of structures. For the structural analysis it has been used the finite elements technique and also the stiffness matrix method. In this case it is found out that the stiffness matrix of an elastic structure depends essentially on E, on the shape and geometrical dimensions of finite elements. The case study from the last part of this paperwork illustrates the random aspect of elasticity modulus.

KEYWORDS: elasticity modulus, homogeneity, structural system, stiffness matrix, variability, finite elements

1. INTRODUCTION

The structural designing is made in the elastic domain, the compulsory stage for all the structural systems, and also in plastic or inelastic domain when the conditions are imposing it. On designing the constructions are determined the displacements, the efforts, the stress and deformation states and it is made an assessment of the safety degree and when it is imposed it is made an assessment of the risk, such as the designing at the action of earthquake. The calculation of the structures in the elastic domain covers a wide area of the designing and from the characteristics of the constitutive materials variation point of view will represent the object of certain investigations, essentially of the manner in which the homogeneity property intervenes.

On designing intervene:

•the structure by its geometrical elements and constitutive material with its properties;

•the actions that can be expressed through their idealization, as distribution and application manner on structure.

The designing in the elastic has certain particularities that allowed it a large utilization. Designing the structures assumes first the transformation of the actions into sectional efforts, for example structures of beam, plates (structural walls) and complex structures case. The structural designing has more levels, most frequent being the designing on the elastic limit and the designing on the ultimate limit strength state.

$$R \geq S \tag{1}$$

where:

R – represents the minimum capacity of strength, which is the elastic limit strength or the ultimate strength in other cases;

S – represents the maximum sectional effect of the loads. It is mentioned the possibility of the existing of more loads hypothesis.

According to developments in this domain, the R and S values are given by statistical and probabilistic distributions that have to be known. A probabilistic designing, although possible in principle, in practice would become very complex and it is not used in it is most general form. In practice it is working with certain formulations that take into account the fact that a large number among the values that intervene on designing have random character.

One first aspect which must be determined in relation (1) is that S represents the sectional effect of loads; thus it assumes the transformation of considered loads into efforts. In this transformation some characteristics of material from structure that are random intervene. Besides, the sectional effect of S loads is affected by the variability of some characteristics of structure, which are not constant values, although it does not present an ideal homogeneity.

In order to achieve the transformation of the loads into efforts are being used the mechanical materials and structures theory lows that are determinist and precise, if the values which intervene are as well determinist. In reality statistical and probabilistic uncertainties intervene.

2. ANALYSIS OF TRANSFORMATION OF THE ACTIONS INTO EFFORTS

In addition to it, we analyze the case of elastic designing namely the transformation of the actions into efforts using the given techniques by the mechanic materials and static of structures, but taking into account the variability of elastic characteristics and the variability of the geometrical values: areas, thickness, moments of inertia and lengths.

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For a structural analysis is used the technique of finite elements and the stiffness matrix method. Using the stiffness matrix method, at static actions, we can reach to a system of linear equations as follows:

$$KD = P \tag{2}$$

where:

K – the stiffness matrix of discretization structure in finite elements;

D – the vector of unknown nodal displacements;

P – the vector of equivalent nodal forces.

The stiffness matrix is obtained through the assembling of the elemental stiffness matrix, of all finite component elements.

The stiffness matrix of finite elements can be determined using the relation:

$$K_e = \iiint_V B^T CBdV \tag{3}$$

The matrix B (x, y) is the geometrical type and depends on the finite element type, the nodal degree of freedom and the shape function of finite element, respectively [1], [2].

C – the constitutive matrix or elasticity matrix which characterizes the elastic properties of material.

The matrix C for the plane finite elements is:

for the plane stress state:

$$C = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \tag{4}$$

for the plane deformation state:

$$C = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \dfrac{\nu}{1-\nu} & 0 \\ \dfrac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \dfrac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{5}$$

In case of the spatial finite elements matrix C can be expressed:

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$$C = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \quad (6)$$

It is noticed that in the general cases the constitutive matrix depends on the elastic characteristics E and ν. In relation (3) the elasticity modulus E considered constantly can be written:

$$C = EC' \quad (7)$$

For a structure formed from the same material, the elasticity modulus E becomes the factor for the stiffness matrix of structure K and can be written EK'.

Because the elasticity modulus E has large values for different construction materials it represents the ponderosity in matrix C. It can be affirmed that the stiffness matrix of an elastic structure depends essentially on E, on the shape and geometrical dimensions of finite elements. It is found out that these values have a random character.

Accordingly, at an accepted load hypothesis expressed through vector P, the vector of nodal displacements D is influenced by the variability of the elasticity modulus, the coefficient of Poisson, in a low extent, and the variability of geometric dimensions.

In case of bars structures, the stiffness of a bar finite element K_e can be written:

$$k_e = \begin{bmatrix} k_{eii} & k_{eij} \\ k_{eji} & k_{ejj} \end{bmatrix} \quad (8)$$

where i and j are ends of bar.

It is found out that the elements of matrixes contain in their terms the elasticity modulus E and geometric elements of bar: the areas of the sections, the central principal inertia moments, the polar inertia moments and the lengths of bars.

Only these characteristics also appear in the assembled stiffness structure matrix. The sectional stiffness of bars can be considered random values and by adequate testing, the obtaining of their statistical distribution could be possible.

3. RANDOM ASPECT OF ELASTICITY MODULUS

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The elasticity modulus as a random value has a normal distribution; the following reasoning are assuming this theory. Relation (2) can be used to determine the nodal displacements of meshed structure and this relation can be written for different situations.

Thus, if the structure is totally realized by the same material such as: steel, reinforced-concrete, wood, then the elasticity modulus used in the deterministic calculation is the same, the matrix K and the equation (2) can be written:

$$K = E \cdot \overline{K} \tag{9}$$

$$KD = E \cdot \overline{K} \cdot D = P \tag{10}$$

The elasticity modulus with a normal distribution is situated in the domain:

$$E_{\min} \le E \le E_{\max} \tag{11}$$

where E_{max} and E_{min} are the maximum values and the minimum values respectively.

For the extremities of the variation interval the equation (19) is:

$$K_{\max} D = E_{\max} \overline{K} D = P \tag{12}$$

$$K_{\min} D = E_{\min} \overline{K} D = P \tag{13}$$

Solving these equations, appropriate displacements can be obtained. For the maximum stiffness are obtained minimum displacements and for the minimum stiffness are obtained maximum displacements:

$$E_{\max} D = \overline{K}^{-1} P \tag{14}$$

$$E_{\min} D = \overline{K}^{-1} P \tag{15}$$

Results:

$$D_{\min} = \frac{1}{E_{\max}} \overline{K}^{-1} P \tag{16}$$

$$D_{\max} = \frac{1}{E_{\min}} \overline{K}^{-1} P \tag{17}$$

On designing and execution of the constructions in which norms are observed, the elasticity modulus of structure is variable and the most proximate value is the medium elasticity modulus.

$$E_{medium} D = \overline{K}^{-1} P \; and \;\; D = \frac{1}{E_{medium}} \overline{K}^{-1} P \tag{18}$$

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It is possible that the real displacements to have the medium value of displacements D_{min} and D_{max} as well.

$$D = \frac{D_{\min} + D_{\max}}{2} = \frac{1}{2}\left(\frac{1}{E_{\min}} + \frac{1}{E_{\max}}\right)\overline{K}^{-1}P \tag{19}$$

$$D = \frac{E_{\max} + E_{\min}}{2E_{\max}E_{\min}}\overline{K}^{-1}P \tag{20}$$

In order to the symmetrical normal distribution the moduli E_{max} and E_{min} can be written:

$$E_{\max} = E_{medium} + \alpha E_{medium} \tag{21}$$

$$E_{\min} = E_{medium} + \alpha E_{medium} \tag{22}$$

and

$$\frac{E_{\max} + E_{\min}}{2E_{\max}E_{\min}} = \frac{1}{E_{medium}(1-\alpha^2)} \tag{23}$$

If the maximum amplitude is admitted to be 20% of the average value (this means an acceptable concentration), results that these ratios $\alpha = \frac{1}{10}$, and $\alpha^2 = \frac{1}{100}$ can be negligible in ratio with unit. This result demonstrates that the medium elasticity modulus is the most appropriate value that should be used in the design. The conclusion is that for designing has to be used the medium value of elasticity modulus, namely:

$$D = \frac{1}{E_{medium}}\overline{K}^{-1}P = K^{-1}P \tag{24}$$

the matrix K being determined with E_{medium} value. If in designing and execution, the real medium values of elasticity modulus and $E_{calculation} < E_{medium\ real}$, are not taken into consideration, this fact is equivalent with the amplification of actions, and the $E_{calculation} > E_{medium\ real}$ is equivalent with the diminishing the actions, the situations more difficult to control can be unfavourable. Actually, some finite elements will have $E < E_{medium}$, and other $E > E_{medium}$, the concentration is around the mean, but due to the symmetry of normal statistical distribution, compensations are produced.

4. CASE STUDY

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It was considered a metallic structure having three opens and five levels (fig. 1) with the load from figure 2. The weight of the structure used in the seismic calculation is G=275323, 38Kg.
Geometrical characteristics:
- for the beams were used profiles IIPE300 with moment of inertia $I=16712cm^4$;

- for the columns were used profiles IIPE400 with moment of inertia $I=46260cm^4$.

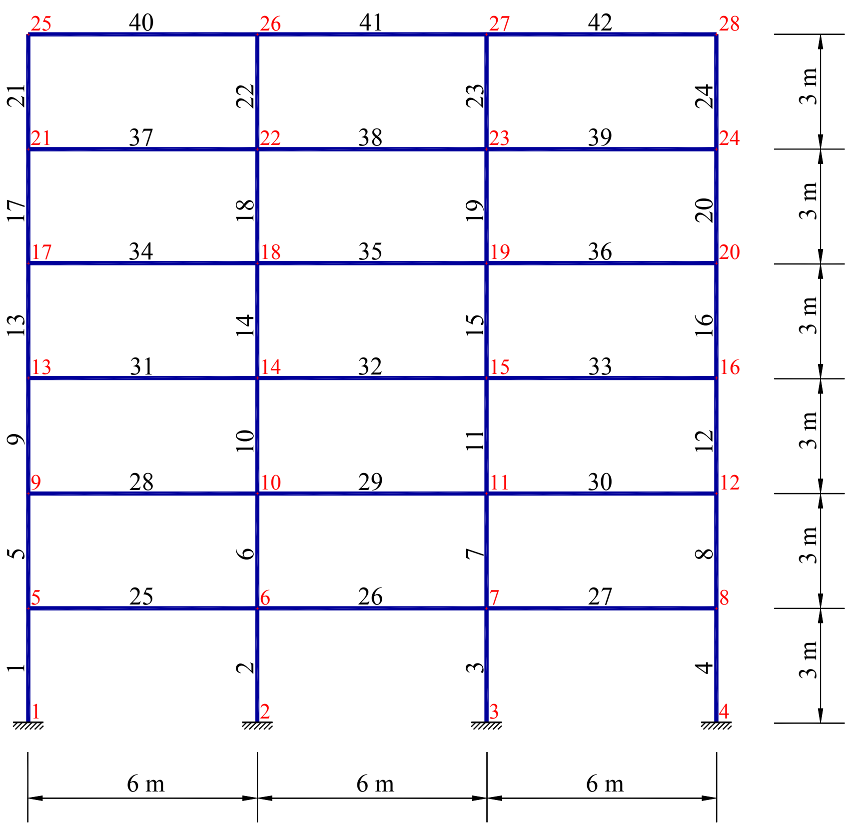

Fig. 1. Geometry of structure

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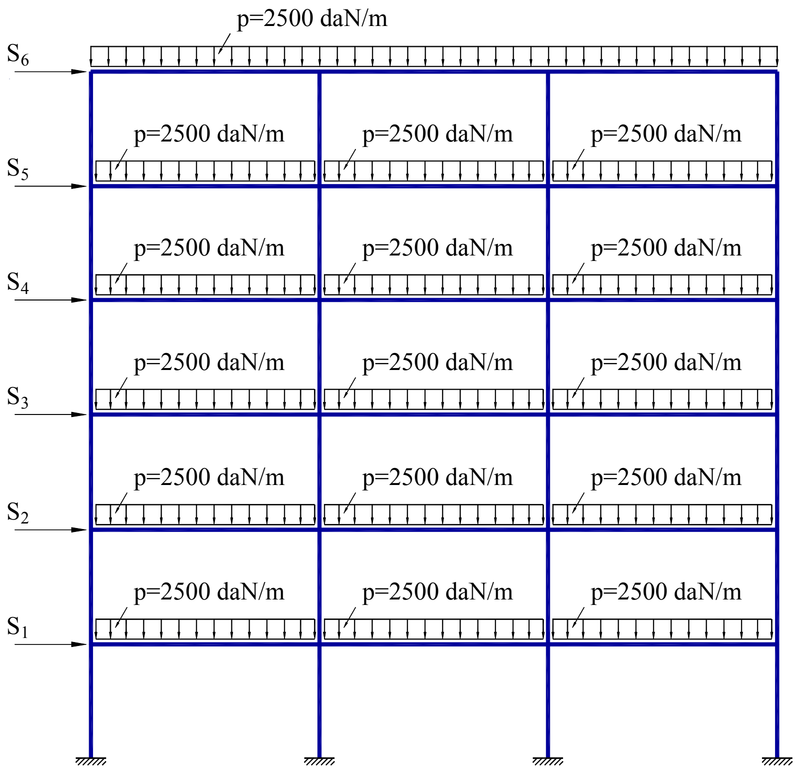

Fig. 2. Loads of structure

Assuming that the structure is in the seismic area B, has the importance class II, the corner period T_c=1s and the ductility factor ψ=0.25, the seismic level forces and the total seismic forces were registered (table 1).

Table 1. Seismic level forces and total seismic forces

	S_1 (daN)	S_2 (daN)	S_3(daN)	S_4(daN)	S_5(daN)	S_6(daN)	S(daN)
E_{medium}	3578.78	56782	7322.12	8730.36	100294	115302	46871.04
E_{min}	3568.46	5623.62	72016	8553.18	9809	11303.16	46061.48
E_{max}	3587.84	5723.22	7422.62	8883.32	102158	11737.04	47570.62
E_{random}	3597.72	56827	7314.47	8721.14	9997.18	115215	46842.93

$E_{medium} = 2.1x10^5 N/mm^2$
$E_{min} = 1.89x10^5 N/mm^2$
$E_{max} = 2.31x10^5 N/mm^2$

A random distribution of elasticity modulus E is presented in table 3 taking into account the number of bars.

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Table 3 E_{random}

E (N/mm^2)	Columns and beams
2.1x10^5	2, 4, 7, 8, 9, 12, 14, 18, 21, 22, 28, 31, 33, 35, 37, 39
1.995x10^5	1, 3, 6, 25, 27, 29
2.205x10^5	10, 11, 19, 26, 34, 36
1.932x10^5	32, 38, 40, 42
2.268x10^5	17, 20, 23
1.89x10^5	5, 15, 16
2.31x10^5	13, 24, 30

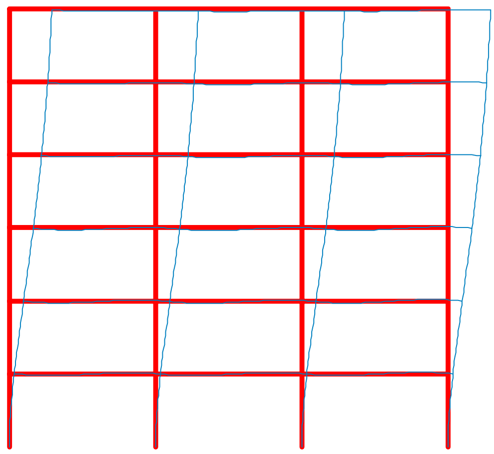

Fig. 3. Deformation of structure

A calculation for four cases it was performed and were written displacements (table 3).

Table 3 Displacements

	D_5 (cm)	D_9 (cm)	D_{13} (cm)	D_{17} (cm)	D_{21} (cm)	D_{25} (cm)
E_{medium}	0.790	2.266	3.784	5.096	077	734
E_{min}	0.8573	2.4583	4.105	5.5278	5911	7.3044
E_{max}	0.7329	2.1018	3.5105	4.7278	5.6375	2465
E_{random}	0.802	2.292	3.8094	5.1234	098	7639

From the performed numerical analyses result a very good approach between the calculated displacements E_{medium} and E_{random}. The difference obtained being under 2%.

The conclusion is that in practice the conformity's verification of medium elasticity modulus achieved with elasticity modulus considered in designing should be a compulsory requirement.

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5. CONCLUSIONS

The variations of elasticity modulus lead to the redistributions of efforts proportional with rapports between elemental and calculation moduli, such as the elements more rigid are loaded more and the elements less rigid are loaded less according to situation in which the elasticity modulus is considered the same.

It is demonstrated that the results of calculation are the most proximate to reality if a medium modulus is used, the using of lower elasticity modulus leading to the equivalent effect with the amplification of actions and the using of a superior modulus leads to an equivalent effect with diminishing of loads.

Accordingly, the conformity's verification of elasticity moduli through designing and execution is necessary.

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