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Design of the spatial structures made of elements with elastic connections in the nodes

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ABSTRACT

The principal elements regarding the design of three dimensional structures through the displacement method on models having elastic jointed bars, are presented. These models are practical for the design of prefabricated structures, in the post-elastic analysis and in the evaluation of the strength capacity of the structures affected by seismic actions.

KEY WORDS: elastic joints, displacement method, rigidity matrix, three-dimensional structures.

1. INTRODUCTION

The models having this type of bars occur in the design of reinforced concrete structures composed of prefabricated elements, also in the postelastic analysis of metallic and reinforced concrete structures. In this last case, through the progression in the elastoplastic domain (metal) or the progressive degradation (reinforced concrete) of certain critical sections (zones), some modifications of the connections take place at the ends of the constitutive elements of the structure. From the point of view of the theoretical analysis, these modifications are reflected in the changing of the rigidity and loading characteristics of the bars, in the changing of the rigidity matrixes of the bars and finally in the changing of the rigidity matrix of the whole structure.

The calculus systemization of these modifications of element rigidity matrixes allows the solving of some concrete problems in the design of new structures, as well as in the expertise of the constructions damaged by earthquakes:

a) The design of the reinforced concrete structures made from prefabricated elements, based on physical models closer to the real prototypes.

b) The improvement of the solutions for some of the structures, through the imposed modification of the connections at the ends of the bars, for

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obtaining some rational distribution of the efforts and the corresponding ductility for the elements and for the entire structure.

c) The biographical analysis of the structures in the initial designing stage, to establish the characteristics of the postelastic adaptability interval and the parameters referring to the reserves of bearing capacity and of postelastic deformability for exceptional loads.

d) The analysis of the earthquake damaged structures, based on physical models established through expertise and thus localizing the degradation zones and evaluating it's degradation degree.

The combined utilization of the calculus ways described at c) and d) allows a more accurate setting of the "nominal assurance degree for seismic actions" (R), defined by the Code P100/92, and represents a way of performing, based on realistic facts, the expertise of the earthquakes damaged structures.

In the following there will be presented the forms for the rigidity matrixes for the bars that are parts of spatial structures and having elastic connections at the ends.

2. THE RIGIDITY MATRIXES OF THE BARS HAVING ELASTIC CONNECTIONS AT THE ENDS THAT ARE PARTS OF A SPATIAL STRUCTURE

The rigidity matrixes are presented in the local coordinate system of the bar, as in Fig. 1 in which are indicated the system of local axes for a bar (ij) having the length L.

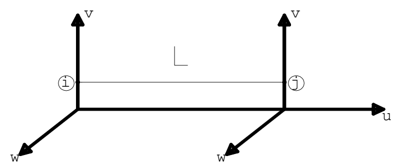

Fig.1

2.1. Defining the degree of fixing at one end of the bar

$$\rho_i = \frac{\theta_i^{(0)} - \theta_i^{(r)}}{\theta_i^{(0)}} \quad (1)$$

where: $\theta_t^{(r)}$ - the real rotation of the bar end

$\theta_t^{(0)}$ - the rotation of the bar end in the hypothesis of perfect pinned connection;

Extreme cases:

$\theta_t^{(r)} = \theta_t^{(0)}$ (hinge connection) for which $\rho_i = 0$

$\theta_t^{(r)} = 0$ (fixed connection) for which $\rho_i = 1$

So, for the ends of the bar with elastic connections (partial fixed connections), the fixed connection grade varies between 0 and 1.

2.2. The elements of the rigidity matrixes of the bars with elastic connections at the ends

The form of a general rigidity matrix for a spatial bar is considered below:

$$[k]_{ij}^{(b)} = \begin{bmatrix} k_{1,1} & 0 & 0 & 0 & 0 & 0 & -k_{1,7} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{2,2} & 0 & 0 & 0 & -k_{2,6} & 0 & k_{2,8} & 0 & 0 & 0 & k_{2,12} \\ 0 & 0 & k_{3,3} & 0 & k_{3,5} & 0 & 0 & 0 & k_{3,9} & 0 & -k_{3,11} & 0 \\ 0 & 0 & 0 & k_{4,4} & 0 & 0 & 0 & 0 & 0 & -k_{4,10} & 0 & 0 \\ 0 & 0 & k_{5,3} & 0 & k_{5,5} & 0 & 0 & 0 & k_{5,9} & 0 & -k_{5,11} & 0 \\ 0 & -k_{6,2} & 0 & 0 & 0 & k_{6,6} & 0 & -k_{6,8} & 0 & 0 & 0 & -k_{6,12} \\ -k_{7,1} & 0 & 0 & 0 & 0 & 0 & k_{7,7} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{8,2} & 0 & 0 & 0 & -k_{8,6} & 0 & k_{8,8} & 0 & 0 & 0 & k_{8,12} \\ 0 & 0 & k_{9,3} & 0 & k_{9,5} & 0 & 0 & 0 & k_{9,9} & 0 & -k_{9,11} & 0 \\ 0 & 0 & 0 & -k_{10,4} & 0 & 0 & 0 & 0 & 0 & k_{10,10} & 0 & 0 \\ 0 & 0 & -k_{11,3} & 0 & -k_{11,5} & 0 & 0 & 0 & -k_{11,9} & 0 & k_{11,11} & 0 \\ 0 & k_{12,2} & 0 & 0 & 0 & -k_{12,6} & 0 & k_{12,8} & 0 & 0 & 0 & k_{12,12} \end{bmatrix} \quad (2)$$

The elements of this rigidity matrix have the following expressions, for each of the 10 types of bars (fig. 2 a÷j).

a) For the bars of types 1-6, the general expressions are:

$$k_{1,1} = k_{7,1} = \frac{G}{L} \cdot F_1^{`}(3);\ k_{1,7} = k_{7,7} = \frac{G}{L} \cdot F_1^{"}(4);$$

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$$k_{2,2}=\frac{4E\cdot I_0^{(v)}}{L}\cdot F_2'(5);\ k_{8,8}=\frac{4E\cdot I_0^{(v)}}{L}\cdot F_2'(6);$$

$$k_{8,2}=\frac{2E\cdot I_0^{(v)}}{L}\cdot F_3'(7);\ k_{2,8}=\frac{2E\cdot I_0^{(v)}}{L}\cdot F_3''(8);$$

$$k_{6,2}=k_{12,2}=\frac{6E\cdot I_0^{(v)}}{L^2}F_4'(9);\ k_{6,8}=k_{12,8}=\frac{6E\cdot I_0^{(v)}}{L^2}F_4'''(10);$$

$$k_{3,3}=\frac{4E\cdot I_0^{(w)}}{L}\cdot F_5'(11);\ k_{9,9}=\frac{4E\cdot I_0^{(w)}}{L}\cdot F_5''(12);$$

$$k_{9,3}=\frac{2E\cdot I_0^{(w)}}{L}\cdot F_6'(13);\ k_{3,9}=\frac{2E\cdot I_0^{(w)}}{L}\cdot F_6''(14);$$

$$k_{5,3}=k_{11,3}=\frac{6E\cdot I_0^{(w)}}{L^2}\cdot F_7'(15);\ k_{5,9}=k_{11,9}=\frac{6E\cdot I_0^{(w)}}{L^2}\cdot F_7''(16);$$

$$k_{4,4}=k_{10,4}=\frac{E}{L}\cdot F_8'(17);\ k_{4,10}=k_{10,10}=\frac{E}{L}\cdot F_8''(18);$$

$$k_{3,5}=k_{9,5}=\frac{6E\cdot I_0^{(w)}}{L^2}\cdot F_7'(19);\ k_{3,11}=k_{9,11}=\frac{6E\cdot I_0^{(w)}}{L^2}\cdot F_7''(20);$$

$$k_{5,5}=k_{11,5}=\frac{12E\cdot I_0^{(w)}}{L^3}\cdot F_9'(21);\ k_{5,11}=k_{11,11}=\frac{12E\cdot I_0^{(w)}}{L^3}\cdot F_9''(22);$$

$$k_{2,6}=k_{8,6}=\frac{6E\cdot I_0^{(v)}}{L^2}\cdot F_4'(23);\ k_{6,12}=k_{8,12}=\frac{6E\cdot I_0^{(v)}}{L^2}\cdot F_4''(24);$$

$$k_{6,6}=k_{12,6}=\frac{12E\cdot I_0^{(v)}}{L^3}\cdot F_{10}'(25);\ k_{6,12}=k_{12,12}=\frac{12E\cdot I_0^{(v)}}{L^3}\cdot F_{10}''(26)$$

The components F1, F2,…, F10 have the expressions centralized in the table 1 for the bars of types 1-6.

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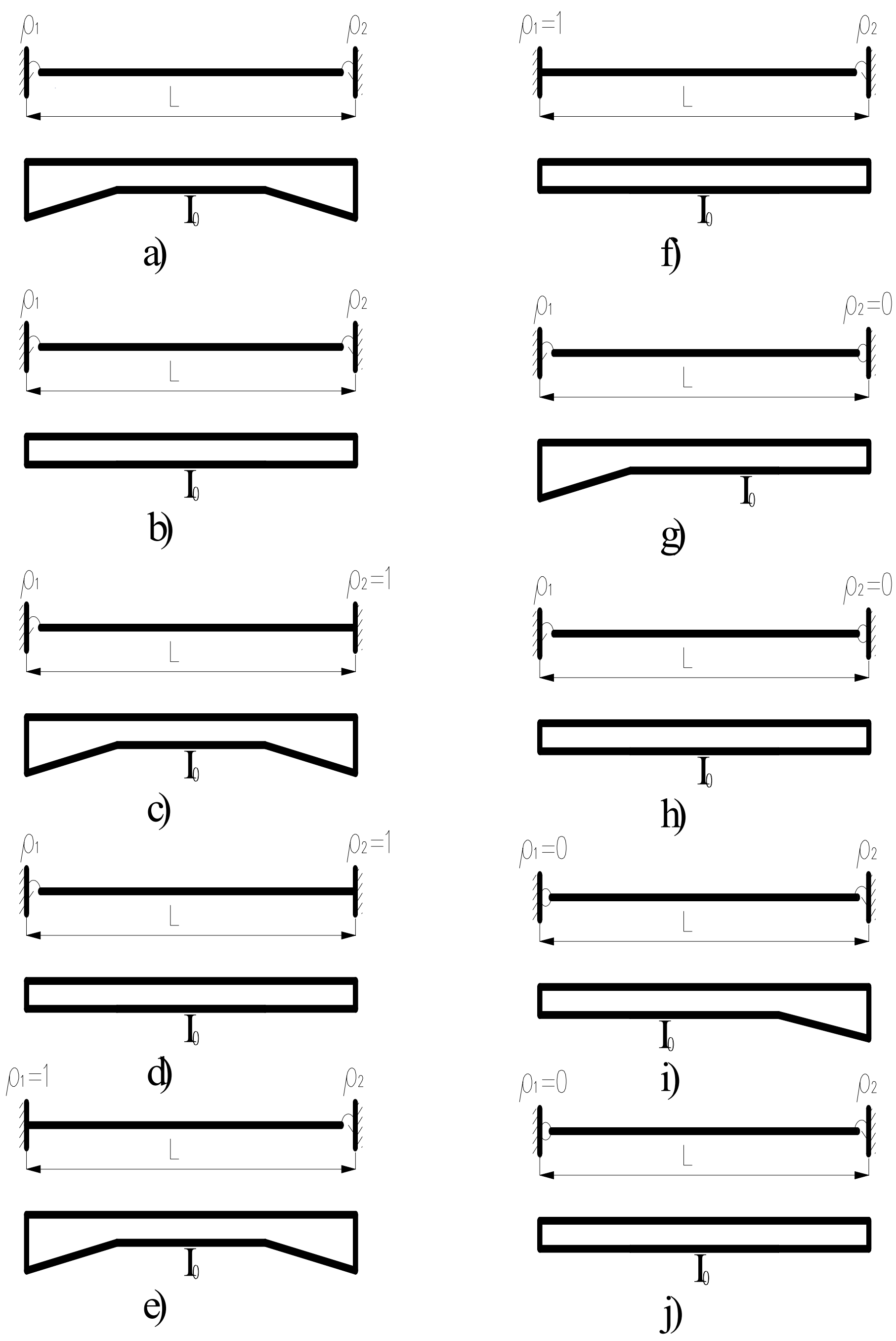

Fig.2

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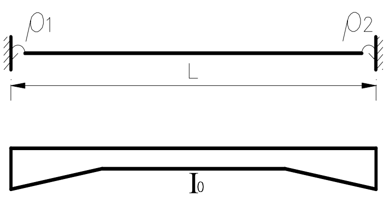
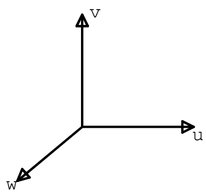
Table 1

BAR 1	$N_1 = 4c'_{(u,w)} c''_{(u,w)} - c^2_{(u,w)} \rho_1^{(u,w)} \rho_2^{(u,w)}$ $N_2 = 4c'_{(u,v)} c''_{(u,v)} - c^2_{(u,v)} \rho_1^{(u,v)} \rho_2^{(u,v)}$
 	$F_1' = I_{tp}^{(u)} \rho_1^{(t)};\ F_1'' = I_{tp}^{(u)} \rho_2^{(t)}$
	$F_2' = 3c''_{(u,w)} \rho_1^{(u,w)} / N_1$
	$F_2'' = 3c'_{(u,w)} \rho_1^{(u,w)} / N_1$
	$F_3' = F_3'' = 3c'_{(u,w)} \rho_1^{(u,w)} \rho_2^{(u,w)} / N_1$
	$F_4' = \rho_1^{(u,w)} \left[2c''_{(u,w)} + c_{(u,w)} \rho_{21}^{(u,w)}\right] / N_1$
	$F_4'' = \rho_2^{(u,w)} \left[2c'_{(u,w)} + c_{(u,w)} \rho_1^{(u,w)}\right] / N_1$
	$F_5' = 3c''_{(u,v)} \rho_1^{(u,v)} / N_2$
	$F_5'' = 3c'_{(u,v)} \rho_2^{(u,v)} / N_2$
	$F_6' = F_6'' = 3c'_{(u,v)} \rho_1^{(u,v)} \rho_2^{(u,v)} / N_2$
	$F_7' = \rho_1^{(u,v)} \left[2c''_{(u,v)} + c_{(u,v)} \rho_2^{(u,v)}\right] / N_2$
	$F_7'' = \rho_2^{(u,v)} \left[2c'_{(u,v)} + c_{(u,v)} \rho_1^{(u,v)}\right] / N_2$
	$F_8' = A_p B_1;\ F_8'' = A_p B_2$
	$F_9' = F_9'' = \left[c'_{(u,v)} \rho_2^{(u,v)} + c''_{(u,v)} \rho_1^{(u,v)} + c_{(u,v)} \rho_1^{(u,v)} \rho_2^{(u,v)}\right] / N_2$
	$F_{10}' = F_{10}'' = \left[c'_{(u,w)} \rho_2^{(u,w)} + c''_{(u,w)} \rho_1^{(u,w)} + c_{(u,w)} \rho_1^{(u,w)} \rho_2^{(u,w)}\right.$

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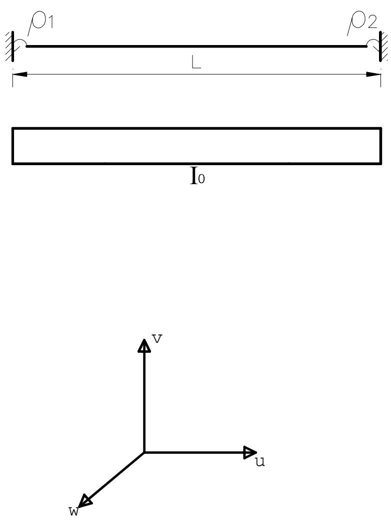
BAR 2	$N_1 = 4 - \rho_1^{(u,w)}\rho_2^{(u,w)}$ $N_2 = 4 - \rho_1^{(u,v)}\rho_2^{(u,v)}$
	$F_1^{`} = I_t^{(u)}\rho_1^{(t)};\ F_1^{"} = I_t^{(u)}\rho_2^{(t)}$
	$F_2^{`} = 3\rho_1^{(u,w)} / N_1 ; F_2^{"} = 3\rho_2^{(u,w)} / N_1$
	$F_3^{`} = F_3^{"} = 3\rho_1^{(u,w)}\rho_2^{(u,w)} / N_1$
	$F_4^{`} = \rho_1^{(u,w)}\left[2 + \rho_2^{(u,w)}\right] / N_1 ;$ $F_4^{"} = \rho_2^{(u,w)}\left[2 + \rho_1^{(u,w)}\right] / N_1$
	$F_5^{`} = 3\rho_1^{(u,v)} / N_2 ; F_5^{"} = 3\rho_2^{(u,v)} / N_2$
	$F_6^{`} = F_6^{"} = 3\rho_1^{(u,v)}\rho_2^{(u,v)} / N_2$
	$F_7^{`} = \rho_1^{(u,v)}\left[2 + \rho_2^{(u,v)}\right] / N_2 ;$ $F_7^{"} = \rho_2^{(u,v)}\left[2 + \rho_1^{(u,v)}\right] / N_2$
	$F_8^{`} = A\ B_1 ;\ F_8^{"} = A\ B_2$
	$F_9^{`} = F_9^{"} = \left[\rho_1^{(u,v)} + \rho_2^{(u,v)} + \rho_1^{(u,v)}\rho_2^{(u,v)}\right] / N_2$
	$F_{10}^{`} = F_{10}^{"} = \left[\rho_1^{(u,w)} + \rho_2^{(u,w)} + \rho_1^{(u,w)}\rho_2^{(u,w)}\right] / N_1$

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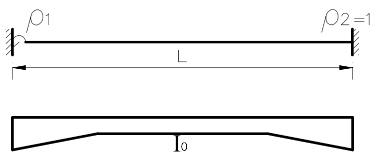
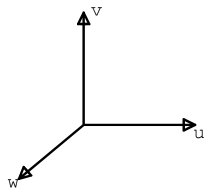
BAR 3	$N_1 = 4c'_{(u,w)} c''_{(u,w)} - c^2_{(u,w)} \rho_1^{(u,w)}$ $N_2 = 4c'_{(u,v)} c''_{(u,v)} - c^2_{(u,v)} \rho_1^{(u,v)}$
	$F'_1 = I_{tp}^{(u)} \rho_1^{(t)};\ F''_1 = I_{tp}^{(u)}$
	$F'_2 = 3c''_{(u,w)} \rho_1^{(u,w)} / N_1;\ F''_2 = 3c'_{(u,w)} / N_1$
	$F'_3 = F''_3 = 3c_{(u,w)} \rho_1^{(u,w)} / N_1$
	$F'_4 = \rho_1^{(u,w)} \left[2c''_{(u,w)} + c_{(u,w)}\right] / N_1$
	$F''_4 = \left[2c'_{(u,w)} + c_{(u,w)} \rho_1^{(u,w)}\right] / N_1$
	$F'_5 = 3c''_{(u,v)} \rho_1^{(u,v)} / N_2;\ F''_5 = 3c'_{(u,v)} / N_2$
	$F'_6 = F''_6 = 3c_{(u,v)} \rho_1^{(u,v)} / N_2$
	$F'_7 = \rho_1^{(u,v)} \left[2c''_{(u,v)} + c_{(u,v)}\right] / N_2$
	$F''_7 = \left[2c'_{(u,v)} + c_{(u,v)} \rho_1^{(u,v)}\right] / N_2$
	$F'_8 = A_p B_1;\ F''_8 = A_p B_2$
	$F'_9 = F''_9 = \left[c'_{(u,v)} + \rho_1^{(u,v)} \left(c''_{(u,v)} + c_{(u,v)}\right)\right] / N_2$
	$F'_{10} = F''_{10} = \left[c'_{(u,w)} + \rho_1^{(u,w)} \left(c''_{(u,w)} + c_{(u,w)}\right)\right] / N_2$

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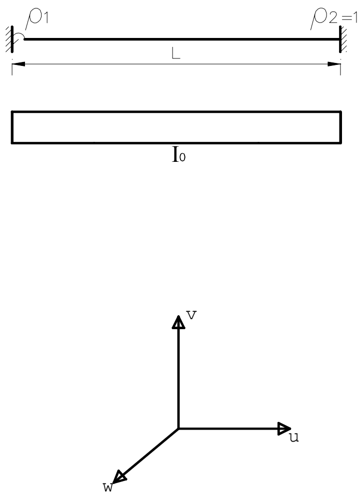
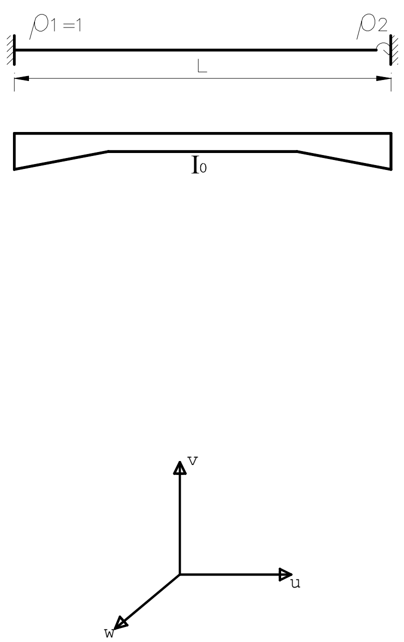
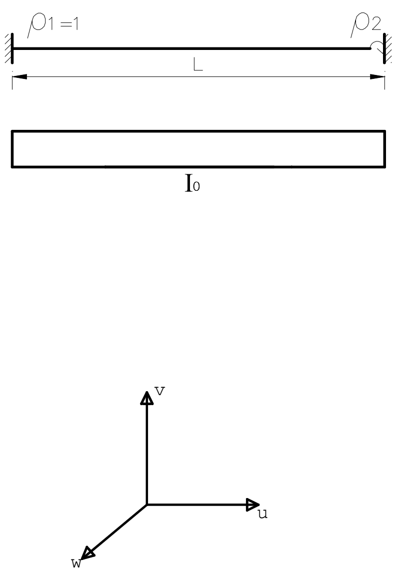
BAR 4	$N_1 = 4 - \rho_1^{(u,w)}$ $N_2 = 4 - \rho_1^{(u,v)}$
	$F_1^{`} = I_t^{(u)} \rho_1^{(u,w)};\ F_1^{"} = I_t^{(u)}$
	$F_2^{`} = 3\rho_1^{(u,w)} / N_1 ; F_2^{"} = 3 / N_1$
	$F_3^{`} = F_3^{"} = 3\rho_1^{(u,w)} / N_1$
	$F_4^{`} = 3\rho_1^{(u,w)} / N_1 ; F_4^{"} = \left[2 + \rho_1^{(u,w)}\right] / N_1$
	$F_5^{`} = 3\rho_1^{(u,v)} / N_2 ; F_5^{"} = 3 / N_2$
	$F_6^{`} = F_6^{"} = 3\rho_1^{(u,v)} / N_2$
	$F_7^{`} = \rho_1^{(u,v)} / N_2 ; F_7^{"} = \left[2 + \rho_1^{(u,v)}\right] / N_2$
	$F_8^{`} = A\ B_1;\ F_8^{"} = A\ B_2$
	$F_9^{`} = F_9^{"} = \left[2\rho_1^{(u,v)} + 1\right] / N_2$
	$F_{10}^{`} = F_{10}^{"} = \left[2\rho_1^{(u,w)} + 1\right] / N_1$

BAR 5	$N_1 = 4c'_{(u,w)}c''_{(u,w)} - c^2_{(u,w)}\rho_2^{(u,w)}$ $N_2 = 4c'_{(u,v)}c''_{(u,v)} - c^2_{(u,v)}\rho_2^{(u,v)}$
	$F_1' = I_{tp}^{(u)};\ F_1'' = I_{tp}^{(u)}\rho_2^{(t)}$
	$F_2' = 3c''_{(u,w)} / N_1 ; F_2'' = 3c'_{(u,w)}\rho_2^{(u,w)} / N_1$
	$F_3' = F_3'' = 3c_{(u,w)}\rho_2^{(u,w)} / N_1$
	$F_4' = \left[2c''_{(u,w)} + c_{(u,w)}\rho_2^{(u,w)}\right] / N_1$
	$F_4'' = \rho_2^{(u,w)}\left[2c'_{(u,w)} + c_{(u,w)}\right] / N_1$
	$F_5' = 3c''_{(u,v)} / N_2 ; F_5'' = 3c'_{(u,v)}\rho_2^{(u,v)} / N_2$
	$F_6' = F_6'' = 3c_{(u,v)}\rho_2^{(u,v)} / N_2$
	$F_7' = \left[2c''_{(u,v)} + c_{(u,v)}\rho_2^{(u,v)}\right] / N_2$
	$F_7'' = \rho_2^{(u,v)}\left[2c'_{(u,v)} + c_{(u,v)}\right] / N_1$
	$F_8' = A_p B_1 ;\ F_8'' = A_p B_2$
	$F_9' = F_9'' = \left[\rho_2^{(u,v)}\left(c'_{(u,v)} + c_{(u,v)}\right) + c''_{(u,v)}\right] / N_2$
	$F_{10}' = F_{10}'' = \left[\rho_2^{(u,w)}\left(c'_{(u,w)} + c_{(u,w)}\right) + c''_{(u,w)}\right] / N_1$

BAR 6	$N_1 = 4 - \rho_2^{(u,w)}$ $N_2 = 4 - \rho_2^{(u,v)}$
	$F_1^{`} = I_t^{(u)};\ F_1^{"} = I_t^{(u)}\rho_2^{(t)}$
	$F_2^{`} = 3/N_1;\ F_2^{"} = 3\rho_2^{(u,w)}/N_1$
	$F_3^{`} = F_3^{"} = 3\rho_2^{(u,w)}/N_1$
	$F_4^{`} = \left[2+\rho_2^{(u,w)}\right]/N_1\ F_4^{"} = 3\rho_2^{(u,w)}/N_2\,;$
	$F_5^{`} = 3/N_2;\ F_5^{"} = 3\rho_2^{(u,v)}/N_2$
	$F_6^{`} = F_6^{"} = 3\rho_2^{(u,v)}/N_2$
	$F_7^{`} = \left[2+\rho_2^{(u,v)}\right]/N_2;\ F_7^{"} = 3\rho_2^{(u,v)}/N_2$
	$F_8^{`} = A\ B_1;\ F_8^{"} = A\ B_2$
	$F_9^{`} = F_9^{"} = \left[2\rho_2^{(u,v)}+1\right]/N_2$
	$F_{10}^{`} = F_{10}^{"} = \left[2\rho_2^{(u,w)}+1\right]/N_2$

b) For the bars of types 7 and 8 elastic-hinged fixed connections, taking into account the hinge from 2 as a spatial hinge $\{\ \rho_2^{(uw)} = \rho_2^{(uv)} = \rho_2^{(t)} = 0\ \}$, the rigidity matrix has dimensions 9 x 9:

$$[k]_{ij}^{(b)} = 0\begin{vmatrix} k_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{2,2} & 0 & 0 & 0 & -k_{2,6} & 0 & 0 & k_{2,9} \\ 0 & 0 & k_{3,3} & 0 & k_{3,5} & 0 & 0 & -k_{3,8} & 0 \\ 0 & 0 & 0 & k_{4,4} & 0 & 0 & -k_{4,7} & 0 & 0 \\ 0 & 0 & k_{5,3} & 0 & k_{5,5} & 0 & 0 & -k_{5,8} & 0 \\ 0 & -k_{6,2} & 0 & 0 & 0 & k_{6,6} & 0 & 0 & -k_{6,9} \\ 0 & 0 & 0 & -k_{7,4} & 0 & 0 & k_{7,7} & 0 & 0 \\ 0 & 0 & -k_{8,3} & 0 & -k_{8,5} & 0 & 0 & k_{8,8} & 0 \\ 0 & k_{9,2} & 0 & 0 & 0 & -k_{9,6} & 0 & 0 & k_{9,9} \end{vmatrix} \quad (27)$$

where:

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$$k_{1,1}=\frac{G}{L}\cdot F_1(28);k_{2,2}=\frac{3EI_0^{(v)}}{L}\cdot F_{11}(29);$$

$$k_{6,2}=k_{9,2}=k_{2,6}=k_{2,9}=\frac{3EI_0^{(v)}}{L^2}\cdot F_{11}(30);$$

$$k_{3,3}=\frac{3EI_0^{(w)}}{L}\cdot F_{12}(31);k_{5,3}=k_{8,3}=k_{3,5}=k_{3,8}=\frac{3EI_0^{(w)}}{L^2}\cdot F_{12}(32);$$

$$k_{4,4}=k_{7,4}=k_{4,7}=k_{7,7}=\frac{E}{L}\cdot F_8(33);k_{5,5}=k_{8,5}=k_{5,8}=k_{8,8}=\frac{3EI_0^{(w)}}{L^3}\cdot F_{12}(34);$$

The components $F_1, F_2, F_{11} si F_{12}$ have the following expressions:

•<u>For the bar with a haunch (bar 7)</u>:

$$F_1=I_t^{(u)}\cdot\rho_1^{(t)};F_8=A_p\cdot B_1(36);F_{1,1}=\frac{\rho_1^{(u,w)}}{c_{(u,w)}^1};F_{1,2}=\frac{\rho_1^{(u,v)}}{c_{(u,v)}^1}(37)$$

- <u>For the bar with constant section (bar 8):</u>

$$F_1=I_t^{(u)}\cdot\rho_1^{(t)}(38);F_8=A\cdot B_1(39);F_{1,1}=\rho_1^{(u,w)}(40);F_{1,2}=\rho_1^{(u,v)}(41)$$

c) <u>For the bars of types 9 and 10</u> (hinged-elastic fixed), taking into account the hinge from 1 as a spatial hinge $\left(\rho_1^{(u,v)}=\rho_1^{(u,w)}=\rho_1^{(t)}=0\right)$, the rigidity matrix has the following form:

$$\begin{array}{cc} & \begin{array}{ccccccccc} u_1=1 & v_1=1 & w_1=1 & \theta_{u2}=1 & \theta_{v2}=1 & \theta_{w2}=1 & u_2=1 & v_2=1 & w_2=1 \end{array} \\ [k]_{ij}^{(b)}= & \left|\begin{array}{ccccccccc}
k_{1,1} & 0 & 0 & 0 & 0 & 0 & -k_{1,7} & 0 & 0\\
0 & k_{2,2} & 0 & 0 & 0 & k_{2,6} & 0 & -k_{2,8} & 0\\
0 & 0 & k_{3,3} & 0 & -k_{3,5} & 0 & 0 & 0 & -k_{3,9}\\
0 & 0 & 0 & k_{4,4} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -k_{5,3} & 0 & k_{5,5} & 0 & 0 & 0 & k_{5,9}\\
0 & k_{6,2} & 0 & 0 & 0 & k_{6,6} & 0 & -k_{6,8} & 0\\
-k_{7,1} & 0 & 0 & 0 & 0 & 0 & k_{7,7} & 0 & 0\\
0 & -k_{8,2} & 0 & 0 & 0 & -k_{8,6} & 0 & k_{8,8} & 0\\
0 & 0 & -k_{9,3} & 0 & k_{9,5} & 0 & 0 & 0 & k_{9,9}
\end{array}\right|(42)\end{array}$$

where:

$$k_{1,1}=k_{7,1}=k_{1,7}=k_{7,7}=\frac{E}{L}\cdot F_8(43);\quad k_{2,2}=k_{8,2}=k_{2,8}=k_{8,6}=\frac{3EI_0^{(w)}}{L^3}\cdot F_{13}(44);$$

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$$k_{6,2}=k_{2,6}=k_{8,6}=k_{6,8}=\frac{3EI_0^{(w)}}{L^3}\cdot F_{13}(45);\quad k_{3,3}=k_{9,3}=k_{3,9}=k_{9,9}=\frac{3EI_0^{(v)}}{L^2}\cdot F_{14}(46);$$

$$k_{4,4}=\frac{G}{L}\cdot F_1(47)\quad;\quad k_{5,5}=\frac{3EI_0^{(v)}}{L}\cdot F_{14}(49);\ k_{6,6}=\frac{3EI_0^{(w)}}{L}\cdot F_{13}(50);$$

$$k_{5,3}=k_{5,3}=k_{9,5}=k_{5,9}=\frac{3EI_0^{(v)}}{L^2}\cdot F_{14}(48)$$

The components F_1, F_8, F_{13}, F_{14}, have the following expressions:

- For a bar with a haunch (bar 9):

$$F_1=I_{ip}^{(u)}\cdot\rho_2^{(t)};\ F_8=A_p\cdot B_2(51);F_{131}=\frac{\rho_2^{(u,w)}}{c''_{(u,w)}};F_{14}=\frac{\rho_2^{(u,v)}}{c''_{(u,v)}}(52)$$

- For the constant section bar (bar 10):

$$F_1=I_t^{(u)}\cdot\rho_2^{(t)}(53);F_8=A\ \cdot B_2(54);F_{13}=\rho_2^{(u,w)}(55);\ \ F_{14}=\rho_2^{(u,v)}(56)$$

Observations:

a) In the previous relations:
A = the area of constant section bar;

A_p = the weighted average area of the bar with haunch section.

unequal haunch: $A_p=\dfrac{A_1a_1+A_0(a_1+2a_0+a_2)+A_2a_2}{2L}$ (57); equal haunch: $A_p=\dfrac{2Aa+A_0(L+a_0)}{2L}(58)$

I_t = the inertial moment at torsion of constant section bar;

I_{tp} = the inertial weighted moment of the bar with haunch.

unequal haunch: $I_{tp}=\dfrac{I_{t1}\cdot a_1+I_{t2}\cdot a_2+I_{t0}(L+a_0)}{2L}$ (59);

equal haunch: $I_{tp}=\dfrac{2I_t\cdot a+I_{t0}(L+a_0)}{2L}(60)$

B= the elastic characteristic of the connection, defined as: $B_{ij} = \frac{u_{ij}}{n_{ij}}(61)$; where:

u_{ij} = the relative axial displacement in node (i) produced by the load $F_i = 1$

n_{ij} = the axial force in node (i) produced by $F_j = 1$

B_{ij} can be determined experimentally or can be approximated with values between 0 and 1.

b) For all the three types of bars, the indexes which affect the inertial moments, the correction coefficients and the fix-connection degrees have the following meanings:

- the indexes that affect the inertial moments $\left(I_t^{(u)}, I_v, I_w\right)$ represent the axes to which the moments of inertia are computed;
- the indexes that affect the correction coefficients for the one haunched bars with $(c^{`}_{(u,w)^{`}}, c^{"}_{(u,w)^{`}}, c_{(u,w)^{`}}, c^{`}_{(u,v)^{`}}, c^{"}_{(u,v)^{`}}, c_{(u,v)^{`}})$ specify the plan in which the
- the indexes of the fix-connection degrees $\left(\rho_1^{(u,w)}, \rho_2^{(u,w)}, \rho_1^{(u,v)}, \rho_2^{(u,v)}\right)$ specify the plan in which the degree of fixing is considered.

c) Regarding the setting of the size of the fixing degrees, the following aspects must be outlined:

- For Rc structures made from prefabricated elements and for analyzing the earthquake-damaged structures, these components are estimated through qualitative measurements, which can take values between 1 and 0 of the fixing degree. It must be outlined, once more, for this stage, the importance of the thoroughly local analysis in place, as well as the qualities regarding the experience and the ability of the specialist, especially in the case of expertise the damaged constructions.
- For the biographic postelastic analysis of the structures, the size of the fix-connection degree can be estimated through the ratios as following:

$$\rho_i^{(e)} = \frac{M_d^{(i)}}{M_u^{(i)}}(62), \text{ where:}$$

$\rho_i^{(e)}$ = *the equivalent fixed-connection degree*;

$M_d^{(i)}$ = *the capable* moment of the critical section (zone) „i", determined taking into account the real state of the damaged section (zone) „i";

$M_u^{(i)}$ = *the ultimate* moment of the critical section (zone) „i" in the undamaged situation.

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